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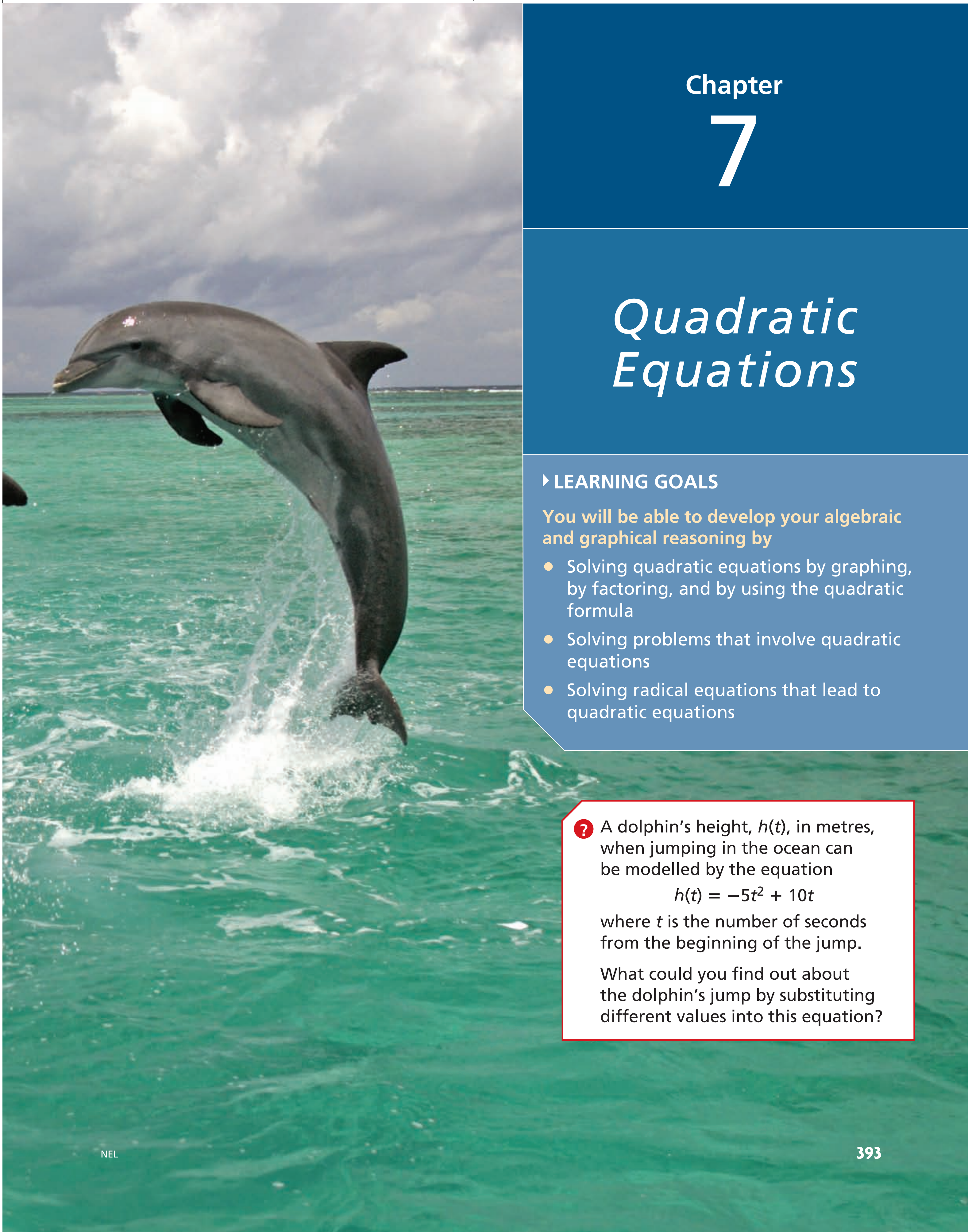

Chapter 7

Quadratic Equations

LEARNING GOALS

You will be able to develop your algebraic and graphical reasoning by

- Solving quadratic equations by graphing, by factoring, and by using the quadratic formula
- Solving problems that involve quadratic equations
- Solving radical equations that lead to quadratic equations

? A dolphin's height, $h(t)$, in metres, when jumping in the ocean can be modelled by the equation

$$h(t) = -5t^2 + 10t$$

where t is the number of seconds from the beginning of the jump.

What could you find out about the dolphin's jump by substituting different values into this equation?

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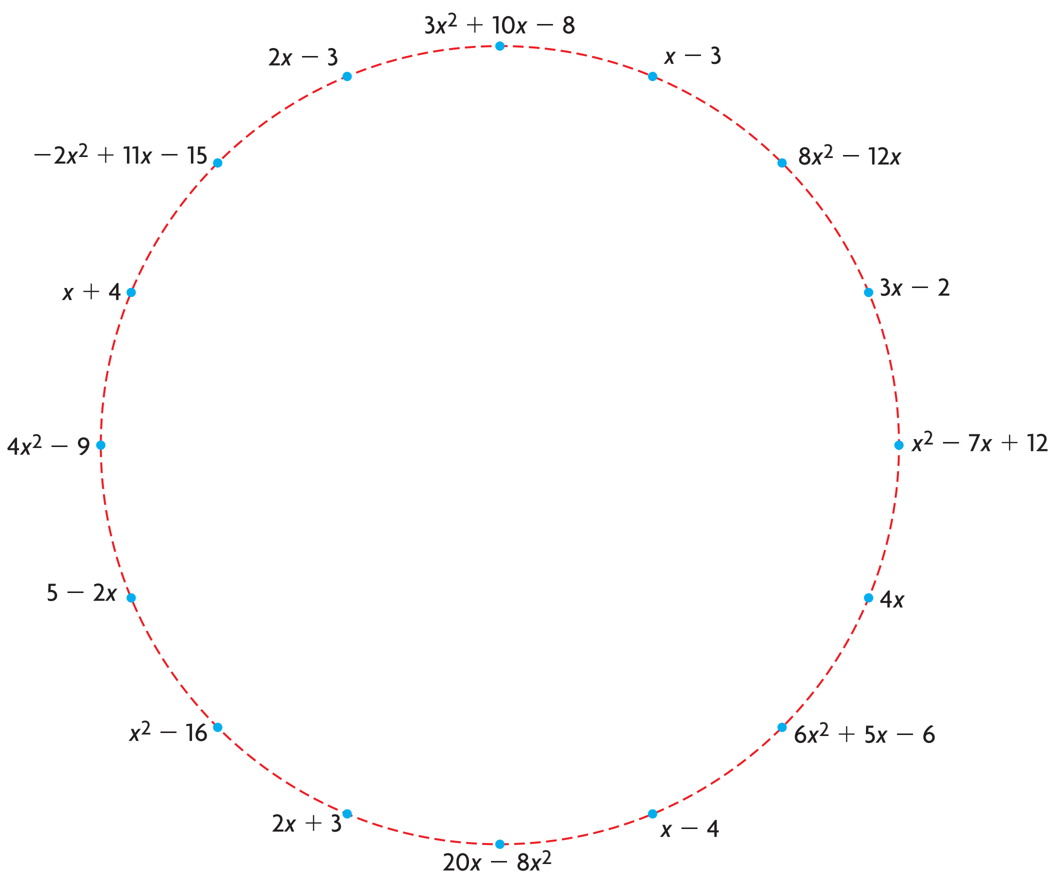
Factoring Design

Jasmine labelled points around a circle with quadratic expressions and their factors, as shown below. Then she made a design by drawing lines to connect each expression to its factors.



? How can you factor each quadratic expression to find out what Jasmine's design looks like?

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A. Draw and label a circle to match Jasmine's circle. Factor each quadratic expression, and record your factors in a table like the one below.

Quadratic Expression	Factor	Factor

B. Join each quadratic expression to its two factors. Use different colours if you want.

C. Compare your design with a classmate's design. Do your designs look the same?

D. Create a design of your own, using at least six quadratic expressions. Exchange designs with a classmate. Was your classmate able to complete your design?

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. A graph is a useful tool to use to solve an equation.
2. The *x*-intercepts or zeros of a function are also solutions to its equation.
3. Since a quadratic function can be written in factored form as a product of two factors, quadratic equations will have two solutions.

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7.1 Solving Quadratic Equations by Graphing

YOU WILL NEED

- graphing technology
- graph paper

GOAL

Solve quadratic equations by graphing the corresponding function.

INVESTIGATE the Math

Bonnie launches a model rocket from the ground with an initial velocity of 68 m/s. The following function, $h(t)$, can be used to model the height of the rocket, in metres, over time, t, in seconds:

$$h(t) = -4.9t^2 + 68t$$

Bonnie's friend Sasha is watching from a lookout point at a safe distance. Sasha's eye level is 72 m above the ground.

? How can you determine the times during the flight when the rocket will be at Sasha's eye level?

A. What is the value of $h(t)$ when the rocket is at Sasha's eye level?

B. Substitute the value of $h(t)$ that you calculated in part A into the function

$$h(t) = -4.9t^2 + 68t$$

to create a **quadratic equation**. You can solve this quadratic equation to determine when the rocket is at Sasha's eye level. Rewrite the quadratic equation in standard form.

C. Graph the function that corresponds to your equation. Use the zeros feature on your calculator to determine the t-intercepts.

D. Graph $h(t) = -4.9t^2 + 68t$. On the same axes, graph the horizontal line that represents Sasha's eye level. Determine the t-coordinates of the points where the two graphs intersect.

E. What do you notice about the t-coordinates of these points?

F. When will the rocket be at Sasha's eye level?

EXPLORE…

- Graph the quadratic function $y = x^2 + 5$. How could you use your graph to solve the equation $21 = x^2 + 5$? What are some other equations you could solve with your graph?

quadratic equation

A polynomial equation of the second degree; the standard form of a quadratic equation is
$ax^2 + bx + c = 0$
For example:
$2x^2 + 4x - 3 = 0$

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Reflecting

G. How were your two graphs similar? How were they different?

H. Describe the two different strategies you used to solve the problem. What are the advantages of each?

APPLY the Math

EXAMPLE 1 Verifying solutions to a quadratic equation

The flight time for a long-distance water ski jumper depends on the initial velocity of the jump and the angle of the ramp. For one particular jump, the ramp has a vertical height of 5.0 m above water level. The height of the ski jumper in flight, $h(t)$, in metres, over time, t, in seconds, can be modelled by the following function:

$$h(t) = 5.0 + 24.46t - 4.9t^2$$

How long does this water ski jumper hold his flight pose?

The skier holds his flight pose until he is 4.0 m above the water.

Olana's Solution

$$h(t) = 5.0 + 24.46t - 4.9t^2$$
$$4.0 = 5.0 + 24.46t - 4.9t^2$$

I substituted 4.0 for $h(t)$ to get a quadratic equation I can use to determine the time when the skier's height above the water is 4.0 m.

$$0 = 1.0 + 24.46t - 4.9t^2$$

I subtracted 4.0 from both sides to put the equation in standard form.

In standard form, $h(t) = 0$. Therefore, the solutions to the equation are the t-intercepts of the graph of this function.

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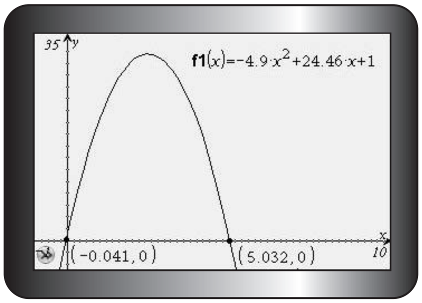

I graphed the function on a calculator. I adjusted the window to show the vertex and the *x*-intercepts. I used the calculator to determine the *x*-intercepts.

The t-intercepts are 5.032 and -0.041.

I reread the problem to make sure each solution made sense. Time can't be negative in this situation, so the jumper did not come out of his pose at -0.041 s. Although $(-0.041, 0)$ is a point on the graph, it doesn't make sense in the context of this problem.

Verify:

$4.0 = 5.0 + 24.46t - 4.9t^2$

$t = 5.032$

LS	RS
4.0	$5.0 + 24.46(5.032) - 4.9(5.032)^2$
	$5.0 + 123.082 \ldots - 124.073 \ldots$
	$4.009 \ldots$

LS = RS

I verified the other solution by substituting it into the original equation. The left side was not quite equal to the right side, but I knew that this was because the calculator is set to show values to three decimal places. The solution is not exact, but it is correct.

The ski jumper holds his flight pose for about 5 s.

Your Turn

Curtis rearranged the equation $4.0 = 5.0 + 24.46t - 4.9t^2$ a different way and got the following equation:

$$4.9t^2 - 24.46t - 1.0 = 0$$

a) Graph the function that is represented by Curtis's equation. How does this graph compare with Olana's graph?

b) Will Curtis get the same solution that Olana did? Explain.

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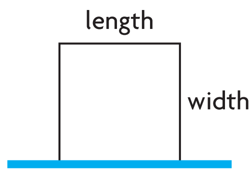
EXAMPLE 2 Graphing to determine the number of roots

Lamont runs a boarding kennel for dogs. He wants to construct a rectangular play space for the dogs, using 40 m of fencing and an existing fence as one side of the play space.



a) Write a function that describes the area, A, in square metres, of the play space for any width, w, in metres.
b) Write equations you could use to determine the widths for areas of 250 m^2, 200 m^2, and 150 m^2.
c) Determine the number of possible widths for each equation using a graph.

Lamont's Solution

Let A represent the area of the play space in square metres.
Let l and w represent the dimensions of the play space in metres.

a) $l + 2w = 40$
$l = 40 - 2w$

From the diagram, I could see that the total length of fencing can be expressed as two widths plus one length. I needed a function that just used variables for area and width, so I rewrote my equation to isolate l.

$$lw = A$$
$$(40 - 2w)w = A$$
$$40w - 2w^2 = A$$

I wrote the formula for the area of the play space and substituted $40 - 2w$ for l. Then I simplified the equation.

b)
$$40w - 2w^2 = 250$$
$$-2w^2 + 40w - 250 = 0$$
$$40w - 2w^2 = 200$$
$$-2w^2 + 40w - 200 = 0$$
$$40w - 2w^2 = 150$$
$$-2w^2 + 40w - 150 = 0$$

To determine the equation for each area, I substituted the area for A. Then I rewrote each quadratic equation in standard form.

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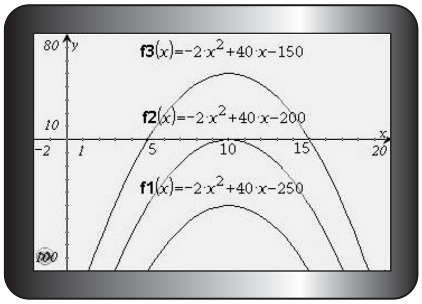
c)



I graphed the corresponding function for each equation.

I can't make a play space with an area of 250 m^2 using 40 m of fencing.

The graph of the first function,

$f_1(w) = -2w^2 + 40w - 250$,

did not cross the w-axis. There are no w-intercepts, so there are no solutions, or **roots**, to the equation.

If I make the play space 10 m wide, the area will be 200 m^2.

The graph of the next function,

$f_2(w) = -2w^2 + 40w - 200$,

intersected the w-axis at its vertex. There is one w-intercept, $w = 10$, so there is one root.

If I make the play space 5 m wide or 15 m wide, the area will be 150 m^2.

The graph of the third function,

$f_3(w) = -2w^2 + 40w - 150$,

has two w-intercepts, $w = 5$ and $w = 15$. This equation has two roots.

roots

The values of the variable that make an equation in standard form equal to zero. These are also called solutions to the equation. These values are also the zeros of the corresponding function and the x-intercepts of its graph.

Your Turn

Is it possible for a quadratic equation to have more than two roots? Use a graph to explain.

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EXAMPLE 3 Solving a quadratic equation in non-standard form

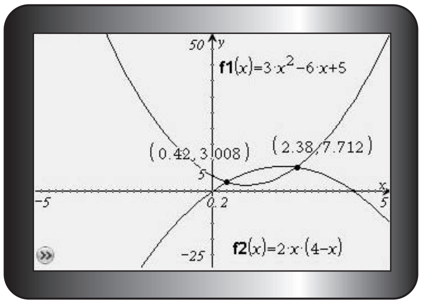
Determine the roots of this quadratic equation. Verify your answers.

$$3x^2 - 6x + 5 = 2x(4 - x)$$

Marwa's Solution

$f(x) = 3x^2 - 6x + 5$
$g(x) = 2x(4 - x)$

I wrote corresponding functions, $f(x)$ and $g(x)$, for each side of the equation to determine the roots.



I graphed each function on a calculator. Then I used the calculator to determine the points of intersection.

The solutions are $x = 0.420$ and $x = 2.380$.

I knew that the solutions of the quadratic equation are the x-coordinates of the points of intersection.

Verify:
$3x^2 - 6x + 5 = 2x(4 - x)$
$x = 0.420$

LS	RS
$3(0.420)^2 - 6(0.420) + 5$	$2(0.420)(4 - 0.420)$
3.009 ...	3.007 ...

LS = RS

I verified the roots by substituting them into the original equation. Both solutions are valid.

Verify:
$3x^2 - 6x + 5 = 2x(4 - x)$
$x = 2.380$

LS	RS
$3(2.380)^2 - 6(2.380) + 5$	$2(2.380)(4 - 2.380)$
7.713 ...	7.711 ...

LS = RS

The roots are $x = 0.420$ and $x = 2.380$.

Your Turn

Rewrite $3x^2 - 6x + 5 = 2x(4 - x)$ in standard form. If you graphed the function that corresponds to your equation in standard form, what x-intercepts would you expect to see? Why?

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In Summary

Key Ideas

- A quadratic equation can be solved by graphing the corresponding quadratic function.
- The standard form of a quadratic equation is

$$ax^2 + bx + c = 0$$

- The roots of a quadratic equation are the x-intercepts of the graph of the corresponding quadratic function. They are also the zeros of the corresponding quadratic function.

Need to Know

- A quadratic equation is any second-degree equation that contains a polynomial in one variable.
- If a quadratic equation is in standard form
  - you can graph the corresponding quadratic function and determine the zeros of the function to solve the equation
- If the quadratic function is not in standard form
  - you can graph the expression on the left side and the expression on the right side as functions on the same axes
  - the x-coordinates of the points of intersection of the two graphs are the roots of the equation
- For any quadratic equation, there can be zero, one, or two real roots. This is because a parabola can intersect the x-axis in zero, one, or two places.

CHECK Your Understanding

1. Solve each equation by graphing the corresponding function and determining the zeros.

a) $2x^2 - 5x - 3 = 0$ **b)** $9x - 4x^2 = 0$

2. Solve each equation by graphing the expressions on both sides of the equation.

a) $x^2 + 5x = 24$ **b)** $0.5x^2 = -2x + 3$

3. Rewrite each equation in standard form. Then solve the equation in standard form by graphing.

a) $6a^2 = 11a + 35$ **b)** $2p^2 + 3p = 1 - 2p$

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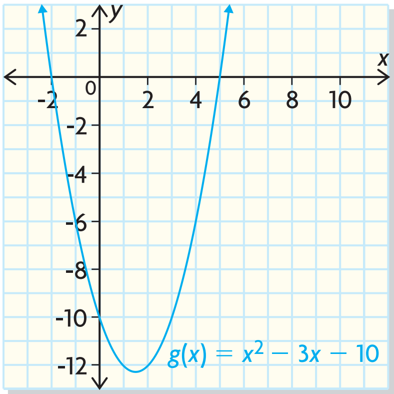
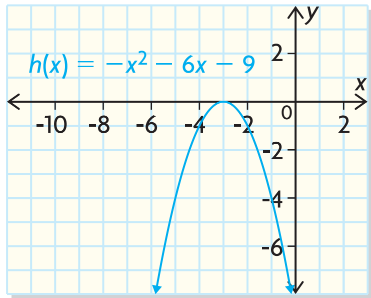
4. For each graph, determine the roots of the corresponding quadratic equation.

a) 

b) 

PRACTISING

5. Solve each equation by graphing the corresponding function and determining the zeros.

a) $3x^2 - 6x - 7 = 0$

b) $0.5z^2 + 3z - 2 = 0$

c) $3b^2 + 8b + 7 = 0$

d) $0.09x^2 + 0.30x + 0.25 = 0$

6. Solve each equation by graphing the expressions on both sides of the equation.

a) $3a^2 = 18a - 21$

b) $5p = 3 - 2p^2$

c) $4x(x + 3) = 3(4x + 3)$

d) $x^2 - 3x - 8 = -2x^2 + 8x + 1$

7. A ball is thrown into the air from a bridge that is 14 m above a river. The function that models the height, $h(t)$, in metres, of the ball over time, t, in seconds is

$$h(t) = -4.9t^2 + 8t + 14$$

a) When is the ball 16 m above the water?

b) When is the ball 12 m above the water? Explain.

c) Is the ball ever 18 m above the water? Explain how you know.

d) When does the ball hit the water?

8. Solve each quadratic equation by graphing.

a) $5x^2 - 2x = 4x + 3$

b) $-2x^2 + x - 1 = x^2 - 3x - 7$

c) $3x^2 - 12x + 17 = -4(x - 2)^2 + 5$

d) $5x^2 + 4x + 3 = -x^2 - 2x$

9. The stopping distance, d, of a car, in metres, depends on the speed of the car, s, in kilometres per hour. For a certain car on a dry road, the equation for stopping distance is

$$d = 0.0059s^2 + 0.187s$$

The driver of the car slammed on his brakes to avoid an accident, creating skid marks that were 120 m long. He told the police that he was driving at the speed limit of 100 km/h. Do you think he was speeding? Explain.

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10. Solve the following quadratic equation using the two methods described below.

$$4x^2 + 3x - 2 = -2x^2 + 5x + 1$$

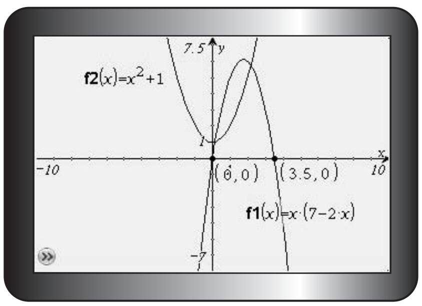
a) Graph the expressions on both sides of the equation, and determine the points of intersection.
b) Rewrite the quadratic equation in standard form, graph the corresponding function, and determine the zeros.
c) Which method do you prefer for this problem? Explain.

11. The length of a rectangular garden is 4 m more than its width. Determine the dimensions of the garden if the area is 117 m^2.

12. Kevin solved the following quadratic equation by graphing the expressions on both sides on the same axes.

$$x(7 - 2x) = x^2 + 1$$

His solutions were $x = 0$ and $x = 3.5$. When he verified his solutions, the left side did not equal the right side.



Verify:
$x(7 - 2x) = x^2 + 1$

$x = 0$

LS	RS
$x(7 - 2x)$	$x^2 + 1$
$(0)(7 - 2(0))$	$(0)^2 + 1$
$(0)(7)$	$0 + 1$
0	1

LS $\neq$ RS

$x = 3.5$

LS	RS
$x(7 - 2x)$	$x^2 + 1$
$(3.5)(7 - 2(3.5))$	$(3.5)^2 + 1$
$(3.5)(7 - 7)$	$12.25 + 1$
0	13.25

LS $\neq$ RS

a) Identify Kevin's error.
b) Determine the correct solution.

13. Solve each equation.
a) $0.25x^2 - 1.48x - 178 = 0$
b) $4.9x(6 - x) + 36 = 2(x + 9) - x^2$

Closing

14. Explain how you could use a graph to determine the number of roots for an equation in the form $ax^2 + bx = c$.

Extending

15. On the same axes, graph these quadratic functions:

$$y = -2x^2 + 20x - 42$$
$$y = x^2 - 10x + 21$$

Write three different equations whose roots are the points of intersection of these graphs.

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7.2 Solving Quadratic Equations by Factoring

GOAL

Solve quadratic equations by factoring.

YOU WILL NEED

- calculator

LEARN ABOUT the Math

The entry to the main exhibit hall in an art gallery is a parabolic arch. The arch can be modelled by the function

$$h(w) = -0.625w^2 + 5w$$

where the height, $h(w)$, and width, w, are measured in feet. Several sculptures are going to be delivered to the exhibit hall in crates. Each crate is a square-based rectangular prism that is 7.5 ft high, including the wheels. The crates must be handled as shown, to avoid damaging the fragile contents.

? What is the maximum width of a 7.5 ft high crate that can enter the exhibit hall through the arch?

EXPLORE…

- What values could you substitute for n and x to make this equation true? $(3x + n)(7x - 7) = 0$

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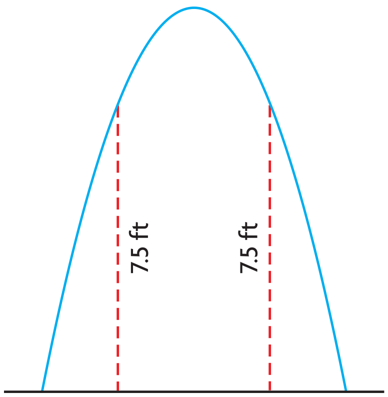
EXAMPLE 1 Solving a quadratic equation by factoring

Determine the distance between the two points on the arch that are 7.5 ft high.

Brooke's Solution



I sketched the situation.

The crate can only fit through the part of the arch that is at least 7.5 ft high. The arch is exactly 7.5 ft high at two points.

The following function describes the arch:
$h(w) = -0.625w^2 + 5w$

The height of the crate is 7.5 ft.

$$7.5 = -0.625w^2 + 5w$$

I wrote an equation, substituting 7.5 for $h(w)$.

$$0.625w^2 - 5w + 7.5 = 0$$

I rewrote the equation in standard form.

I decided to subtract $-0.625w^2 + 5w$ from both sides so the coefficient of w^2 would be positive.

$$\frac{0.625w^2}{0.625} - \frac{5w}{0.625} + \frac{7.5}{0.625} = \frac{0}{0.625}$$

I divided by 0.625 to simplify the equation.

$$w^2 - 8w + 12 = 0$$
$$(w - 2)(w - 6) = 0$$

I factored the equation.

$$w - 2 = 0 \quad \text{or} \quad w - 6 = 0$$
$$w = 2 \qquad\qquad w = 6$$

If the product of two factors is 0, then at least one factor must equal 0.

The parabola reaches a height of exactly 7.5 ft at widths of 2 ft and 6 ft.

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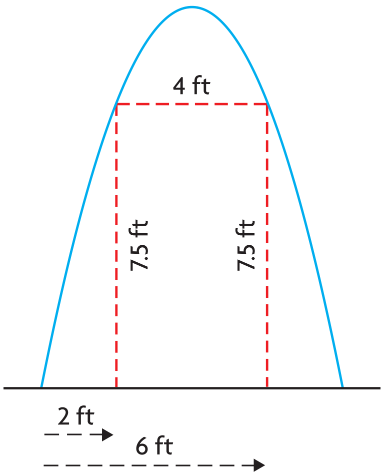
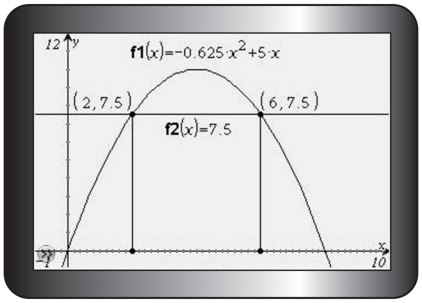


I determined the difference between the widths to determine the maximum width of the crate.

To fit through the archway, the crate cannot be more than 4 ft wide.



I checked my solution by graphing $y = -0.625x^2 + 5x$ and $y = 7.5$.

The x-coordinates of the points of intersection are 2 and 6, so my solution is correct.

Reflecting

A. How did rewriting the equation in standard form and then factoring it help Brooke determine the roots?

B. Was Brooke's decision to divide both sides of the equation by 0.625 (the coefficient of w^2) reasonable? Explain.

C. Describe another way that Brooke could verify the solutions to her equation.

D. Tim says that if you know the roots of an equation, you can use factors to determine the equation. How could Tim use the roots 2 and 6 to determine the equation that Brooke solved?

E. Can you always use factoring to solve a quadratic equation? Explain.

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APPLY the Math

EXAMPLE 2 Solving a quadratic equation using a difference of squares

Determine the roots of the following equation:

$$75p^2 - 192 = 0$$

Verify your solution.

Alberto's Solution

$$75p^2 - 192 = 0$$
$$\frac{75p^2}{3} - \frac{192}{3} = \frac{0}{3}$$
$$25p^2 - 64 = 0$$
$$(5p - 8)(5p + 8) = 0$$

I noticed that 3 is a factor of both 75 and 192.

I noticed that $25p^2$ and 64 are both perfect squares, so $25p^2 - 64$ is a difference of squares.

$5p - 8 = 0$ or $5p + 8 = 0$

$5p = 8$ $\quad$ $5p = -8$

$p = \frac{8}{5}$ $\quad$ $p = -\frac{8}{5}$

I determined the roots.

The roots are $\frac{8}{5}$ and $-\frac{8}{5}$.

$$75p^2 - 192 = 0$$
$$75p^2 = 192$$
$$p^2 = \frac{192}{75}$$
$$p^2 = \frac{64}{25}$$
$$p = \pm\sqrt{\frac{64}{25}}$$
$$p = \pm\frac{8}{5}$$

I decided to verify my solutions by solving the equation using a different method.

I isolated p^2 and then took the square root of each side. I knew that p^2 has two possible square roots, one positive and the other negative.

My solution matched the solution I obtained by factoring.

Your Turn

How can you tell that any equation with a difference of squares is factorable? What can you predict about the roots?

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EXAMPLE 3 Solving a quadratic equation with only one root

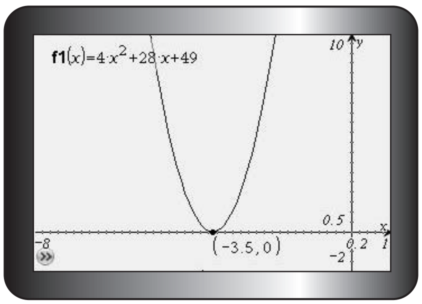
Solve and verify the following equation:

$$4x^2 + 28x + 49 = 0$$

Arya's Solution

$$\begin{aligned} 4x^2 + 28x + 49 &= 0 \\ (2x + 7)(2x + 7) &= 0 \\ 2x + 7 &= 0 \\ x &= -3.5 \end{aligned}$$

I factored the trinomial. I noticed that both factors are the same, so there is only one root.



I decided to verify my solution by graphing the corresponding quadratic function.

I noticed that the vertex of the function is on the x-axis at -3.5, so my solution makes sense.

Your Turn

How can factoring an equation help you determine whether the equation has two roots or one root?

EXAMPLE 4 Using reasoning to write an equation from its roots

Tori says she solved a quadratic equation by graphing. She says the roots were -5 and 7. How can you determine an equation that she might have solved?

Philip's Solution

$x = -5$ or $x = 7$

The x-intercepts of the quadratic function are the roots of the equation.

$x + 5 = 0$ $\quad$ $x - 7 = 0$

One factor is $x + 5$.

The other factor is $x - 7$.

I decided to use the roots to help me write the factors of the equation.

$$\begin{aligned} (x + 5)(x - 7) &= 0 \\ x^2 + 5x - 7x - 35 &= 0 \\ x^2 - 2x - 35 &= 0 \end{aligned}$$

I wrote the factors as a product. Since each root is equal to 0, their product is also equal to 0.

I simplified to write the equation in standard form.

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Your Turn

The x-intercepts of the graph of a quadratic function are 3 and -2.5. Write a quadratic equation that has these roots.

EXAMPLE 5 Describing errors in a solution

Matthew solved a quadratic equation as shown. Identify and correct the error in Matthew's solution.

$$4x^2 = 9x$$
$$\frac{4x^2}{x} = \frac{9x}{x}$$
$$4x = 9$$
$$x = 2.25$$

Raj's Solution

Matthew made an error in the second line of his solution. When he divided both sides by x, he eliminated a possible factor, $x = 0$.

$\frac{4x^2}{x}$ and $\frac{9x}{x}$ are not defined when $x = 0$, so Matthew cannot divide by x.

Correctly solving the equation:

$$4x^2 - 9x = 0$$
$$x(4x - 9) = 0$$

To solve the equation, I rewrote it in standard form and then factored the left side.

$x = 0$ or $4x - 9 = 0$
$x = 0$ or $4x = 9$
$x = 2.25$

For my equation to be true, either x or $4x - 9$ must equal 0.

Verify:
$4x^2 - 9x = 0$

$x = 0$

LS	RS
$4x^2 - 9x$	0
$4(0)^2 - 9(0)$	
$0 - 0$	
0	

LS = RS

$x = 2.25$

LS	RS
$4x^2 - 9x$	0
$4(2.25)^2 - 9(2.25)$	
$20.25 - 20.25$	
0	

LS = RS

I verified each solution by substituting it into the original equation. For both solutions, the left side is equal to the right side. Therefore, both solutions are correct.

Your Turn

What number will always be a root of an equation that can be written in standard form as $ax^2 + bx = 0$? Explain how you know.

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In Summary

Key Idea

- Some quadratic equations can be solved by factoring.

Need to Know

- To factor an equation, start by writing the equation in standard form.
- You can set each factor equal to zero and solve the resulting linear equations. Each solution is a solution to the original equation.
- If the two roots of a quadratic equation are equal, then the quadratic equation is said to have one solution.

CHECK Your Understanding

1. Solve by factoring. Verify each solution.

a) $x^2 - 11x + 28 = 0$
b) $x^2 - 7x - 30 = 0$
c) $2y^2 + 11y + 5 = 0$
d) $4t^2 + 7t - 15 = 0$

2. Solve by factoring.

a) $x^2 - 121 = 0$
b) $9r^2 - 100 = 0$
c) $x^2 - 15x = 0$
d) $3y^2 + 48y = 0$
e) $s^2 - 12s + 36 = 0$
f) $16p^2 + 8p + 1 = 0$
g) $-14z^2 + 35z = 0$
h) $5q^2 - 9q = 0$

PRACTISING

3. Solve by factoring. Verify each solution.

a) $x^2 - 9x - 70 = 0$
b) $x^2 + 19x + 48 = 0$
c) $3a^2 + 11a - 4 = 0$
d) $6t^2 - 7t - 20 = 0$

4. Solve each equation.

a) $12 - 5x = 2x^2$
b) $4x^2 = 9 - 9x$
c) $49d^2 + 9 = -42d$
d) $169 = 81g^2$

5. Geeta solved this equation:

$$20x^2 - 21x - 27 = 0$$

Her solutions were $x = 0.75$ and $x = -1.8$.

a) Factor and solve the equation.
b) What error do you think Geeta made?

6. Determine the roots of each equation.

a) $5u^2 - 10u - 315 = 0$
b) $0.25x^2 + 1.5x + 2 = 0$
c) $1.4y^2 + 5.6y - 16.8 = 0$
d) $\frac{1}{2}k^2 + 5k + 12.5 = 0$

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7. The graph of a quadratic function has x-intercepts -5 and -12. Write a quadratic equation that has these roots.

8. A bus company charges \$2 per ticket but wants to raise the price. The daily revenue that could be generated is modelled by the function

$$R(x) = -40(x - 5)^2 + 25\ 000$$

where x is the number of 10¢ price increases and $R(x)$ is the revenue in dollars. What should the price per ticket be if the bus company wants to collect daily revenue of \$21 000?

9. Solve and verify the following equation:

$$5x - 8 = 20x^2 - 32x$$

10. Identify and correct any errors in the following solution:

$$\begin{aligned} 5a^2 - 100 &= 0 \\ 5a^2 &= 100 \\ a^2 &= 25 \\ \sqrt{a^2} &= \sqrt{25} \\ a &= 5 \end{aligned}$$

11. Identify and correct the errors in this solution:

$$\begin{aligned} 4r^2 - 9r &= 0 \\ (2r - 3)(2r + 3) &= 0 \end{aligned}$$

$$\begin{aligned} 2r - 3 &= 0 \quad \text{or} \quad & 2r + 3 &= 0 \\ 2r &= 3 & 2r &= -3 \\ r &= 1.5 \quad \text{or} & r &= -1.5 \end{aligned}$$

12. **a)** Write a quadratic function with zeros at 0.5 and -0.75.
b) Compare your function with a classmate's function. Did you get the same function?
c) Working with a classmate, determine two other possible functions with the same zeros.

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13. Sanela sells posters to stores. The profit function for her business is

$$P(n) = -0.25n^2 + 6n - 27$$

where n is the number of posters sold per month, in hundreds, and $P(n)$ is the profit, in thousands of dollars.

a) How many posters must Sanela sell per month to break even?

b) If Sanela wants to earn a profit of \$5000 ($P(n) = 5$), how many posters must she sell?

c) If Sanela wants to earn a profit of \$9000, how many posters must she sell?

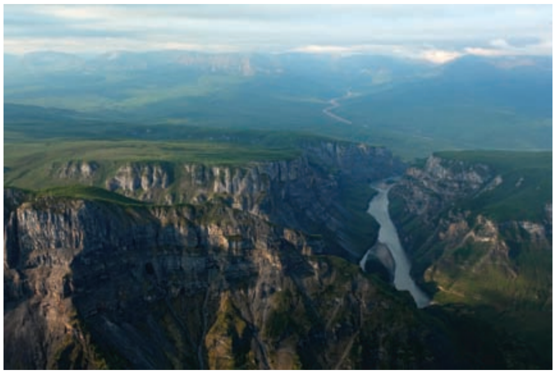
d) What are the domain and range of the profit function? Explain your answer.

14. Samuel is hiking along the top of First Canyon on the South Nahanni River in the Northwest Territories. When he knocks a rock over the edge, it falls into the river, 1260 m below. The height of the rock, $h(t)$, at t seconds, can be modelled by the following function:

$$h(t) = -25t^2 - 5t + 1260$$

a) How long will it take the rock to reach the water?

b) What is the domain of the function? Explain your answer.



The Nahanni River in the Northwest Territories is a popular area for wilderness tours and whitewater rafting.

15. a) Create a quadratic equation that can be solved by factoring. Exchange equations with a classmate, and solve each other's equations.

b) Modify the equation you created so that it cannot be factored. Explain how you modified it. Then exchange equations with a classmate again, and solve each other's equations a different way.

Closing

16. a) Explain the steps you would follow to solve a quadratic equation by factoring.

b) When does it make sense to solve a quadratic equation by factoring? When does it make sense to use graphing?

Extending

17. One root of an equation in the form $ax^2 + c = 0$ is 6.

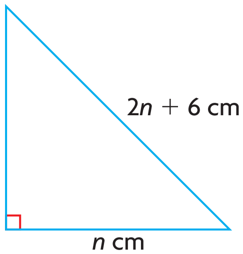
a) What can you predict about the factors if there is no bx term in the equation?

b) Determine the other root.

c) Write the equation in factored form.

d) Write the equation in standard form.

18. The perimeter of this right triangle is 60 cm. Determine the lengths of all three sides.



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7.3 Solving Quadratic Equations Using the Quadratic Formula

YOU WILL NEED

- graphing technology

GOAL

Use the quadratic formula to determine the roots of a quadratic equation.

EXPLORE...

- Kyle was given the following function:

  $y = 2x^2 + 12x - 14$

  He wrote it in vertex form:

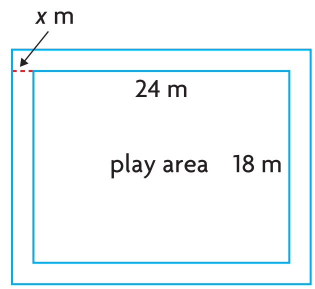
  $y = 2(x + 3)^2 - 32$

  How can you use the vertex form to solve this equation?

  $2x^2 + 12x - 14 = 0$

LEARN ABOUT the Math

Ian has been hired to lay a path of uniform width around a rectangular play area, using crushed rock. He has enough crushed rock to cover 145 m^2.



? If Ian uses all the crushed rock, how wide will the path be?

EXAMPLE 1 Using the quadratic formula to solve a quadratic equation

Determine the width of the path that will result in an area of 145 m^2.

Alima's Solution

Area of border = Total area − Play area
The play area is a constant, (length)(width) or (24 m)(18 m) or 432 m^2.

The total area of the playground, P, can be represented as

$P = \text{(length)(width)}$
$P = (2x + 24)(2x + 18)$

I wrote a function that describes how the area of the path, A square metres, changes as the width of the path, x metres, changes.

The area of the path, $A(x)$, can be represented as

$A(x) = (2x + 24)(2x + 18) - 432$
$A(x) = 4x^2 + 84x + 432 - 432$
$A(x) = 4x^2 + 84x$

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$145 = 4x^2 + 84x$

I substituted the area of 145 m^2 for $A(x)$.

$4x^2 + 84x - 145 = 0$
$a = 4$, $b = 84$, and $c = -145$

I rewrote the equation in standard form:

$ax^2 + bx + c = 0$

Then I determined the values of the coefficients a, b, and c.

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-84 \pm \sqrt{84^2 - 4(4)(-145)}}{2(4)}$$

$$x = \frac{-84 \pm \sqrt{9376}}{8}$$

$$x = \frac{-84 + \sqrt{9376}}{8} \quad \text{or} \quad x = \frac{-84 - \sqrt{9376}}{8}$$

$$x = 1.603 \ldots \quad \text{or} \quad x = -22.603 \ldots$$

The **quadratic formula** can be used to solve any quadratic equation. I wrote the quadratic formula and then substituted the values of a, b, and c from my equation into the formula.

I simplified the right side.

I separated the quadratic expression into two solutions.

quadratic formula
A formula for determining the roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$; the quadratic formula is written using the coefficients of the variables and the constant in the quadratic equation that is being solved:

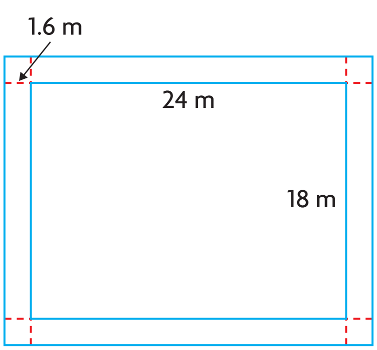
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is derived from $ax^2 + bx + c = 0$ by isolating x.

The solution -22.603 is inadmissible.

I knew that the width of the path couldn't be negative, so $-22.603 \ldots$ is an **inadmissible solution**.

inadmissible solution
A root of a quadratic equation that does not lead to a solution that satisfies the original problem.



I sketched the path and verified my solution by determining the area of the path. To do this, I added the areas of all the rectangles that make up the path.

$18(1.603 \ldots) = 28.866 \ldots \text{ m}^2$
$24(1.603 \ldots) = 38.489 \ldots \text{ m}^2$
$(1.603 \ldots)(1.603 \ldots) = 2.571 \ldots \text{ m}^2$

Area of path $= 2(28.866 \ldots) + 2(38.489 \ldots) + 4(2.571 \ldots)$
Area of path $= 144.999 \ldots \text{ m}^2$

The total area is very close to 145 m^2.
The path should be about 1.6 m wide.

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Reflecting

A. Why did Alima need to write her equation in standard form?

B. Which part of the quadratic formula shows that there are two possible solutions?

C. Why did Alima decide not to use the negative solution?

D. In this chapter, you have learned three methods for solving quadratic equations: graphing, factoring, and using the quadratic formula. What are some advantages and disadvantages of each method?

APPLY the Math

EXAMPLE 2 Connecting the quadratic formula to factoring

Solve the following equation:

$$6x^2 - 3 = 7x$$

Adrianne's Solution

$$6x^2 - 3 = 7x$$
$$6x^2 - 7x - 3 = 0$$
$$a = 6,\ b = -7, \text{ and } c = -3$$

First, I rewrote the equation in standard form to determine the values of *a*, *b*, and *c*.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$
$$x = \frac{7 \pm \sqrt{121}}{12}$$
$$x = \frac{7 \pm 11}{12}$$

I wrote the quadratic formula and substituted the values of *a*, *b*, and *c*.

I simplified the right side. I realized that 121 is a perfect square.

$$x = \frac{18}{12} \quad \text{or} \quad x = \frac{-4}{12}$$
$$x = \frac{3}{2} \quad \text{or} \quad x = \frac{-1}{3}$$

I determined the two solutions.

Verify:

$$6x^2 - 7x - 3 = 0$$
$$(3x + 1)(2x - 3) = 0$$

$$3x + 1 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$3x = -1 \qquad\qquad 2x = 3$$
$$x = \frac{-1}{3} \qquad\qquad x = \frac{3}{2}$$

The solutions match those I got using the quadratic formula.

If the radicand in the quadratic formula is a perfect square, then the original equation can be factored. I decided to verify my solution by factoring the original equation.

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Your Turn

Sandy was given the following equation:

$$12x^2 - 47x + 45 = 0$$

She used the quadratic formula to solve it.

$$x = \frac{-(-47) \pm \sqrt{(-47)^2 - 4(12)(45)}}{2(12)}$$

$$x = \frac{47 \pm \sqrt{49}}{24}$$

$$x = 2\frac{1}{4} \quad \text{or} \quad x = \frac{5}{3}$$

Could Sandy use factoring to verify her solutions? Explain how you know.

EXAMPLE 3 Determining the exact solution to a quadratic equation

Solve this quadratic equation:

$$2x^2 + 8x - 5 = 0$$

State your answer as an exact value.

Quyen's Solution

$2x^2 + 8x - 5 = 0$
$a = 2$, $b = 8$, and $c = -5$

The equation was in standard form. I determined the values of *a*, *b*, and *c*.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{8^2 - 4(2)(-5)}}{2(2)}$$

I wrote the quadratic formula and substituted the values of *a*, *b*, and *c*.

$$x = \frac{-8 \pm \sqrt{104}}{4}$$

$$x = \frac{-8 \pm \sqrt{4}\sqrt{26}}{4}$$

$$x = \frac{-8 \pm 2\sqrt{26}}{4}$$

$$x = \frac{-4 \pm \sqrt{26}}{2}$$

I simplified the expression.

I noticed that one factor of 104 is 4, which is a perfect square. I simplified the radical.

I simplified the fraction.

$$x = \frac{-4 + \sqrt{26}}{2} \quad \text{or} \quad x = \frac{-4 - \sqrt{26}}{2}$$

Another way to write my solution is to show two separate values.

Your Turn

Solve the following quadratic equation:

$$5x^2 - 10x + 3 = 0$$

State your answer as an exact value.

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EXAMPLE 4 Solving a pricing problem

A store rents an average of 750 video games each month at the current rate of \$4.50. The owners of the store want to raise the rental rate to increase the revenue to \$7000 per month. However, for every \$1 increase, they know that they will rent 30 fewer games each month. The following function relates the price increase, p, to the revenue, r:

$$(4.5 + p)(750 - 30p) = r$$

Can the owners increase the rental rate enough to generate revenue of \$7000 per month?

Christa's Solution

$$(4.5 + p)(750 - 30p) = r$$
$$3375 + 615p - 30p^2 = r$$
$$3375 + 615p - 30p^2 = 7000$$
$$-30p^2 + 615p + 3375 = 7000$$
$$-30p^2 + 615p - 3625 = 0$$
$$\frac{-30p^2}{-5} + \frac{615p}{-5} - \frac{3625}{-5} = \frac{0}{-5}$$
$$6p^2 - 123p + 725 = 0$$

I simplified the function.

I substituted the revenue of \$7000 for r and wrote the equation in standard form.

I divided each term by -5 to simplify the equation.

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$p = \frac{-(-123) \pm \sqrt{(-123)^2 - 4(6)(725)}}{2(6)}$$
$$p = \frac{123 \pm \sqrt{-2271}}{12}$$

I didn't try to factor the equation since the numbers were large. I decided to use the quadratic formula.

I substituted the values of a, b, and c into the quadratic formula.

I simplified the right side.

$\sqrt{-2271}$ is not a real number, so there are no real solutions to this equation. It is not possible for the store to generate revenue of \$7000 per month by increasing the rental rate.

I noticed that the radicand is negative.

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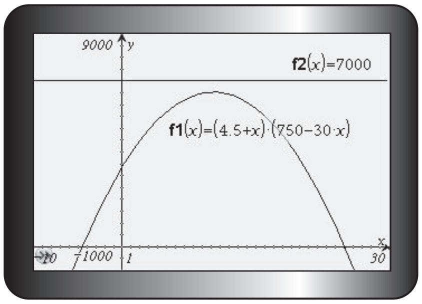

To verify my answer, I graphed
$y = (4.5 + x)(750 - 30x)$ and $y = 7000$
There is no point of intersection.

Your Turn

Is it possible for the store to generate revenue of \$6500 per month by increasing the rental rate? Explain.

In Summary

Key Idea

- The roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be determined by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Need to Know

- The quadratic formula can be used to solve any quadratic equation, even if the equation is not factorable.
- If the radicand in the quadratic formula simplifies to a perfect square, then the equation can be solved by factoring.
- If the radicand in the quadratic formula simplifies to a negative number, then there is no real solution for the quadratic equation.

CHECK Your Understanding

1. Solve each equation using the quadratic formula. Verify by graphing.

a) $x^2 + 7x - 5 = 0$
b) $8x^2 + 35x + 12 = 0$
c) $2a^2 - 5a + 1 = 0$
d) $-20p^2 + 7p + 3 = 0$

2. Solve each equation using the quadratic formula.

a) $x^2 + 5x - 6 = 0$
b) $4x + 9x^2 = 0$
c) $25x^2 - 121 = 0$
d) $12x^2 - 17x - 40 = 0$

3. Solve each equation in question 2 by factoring. Which method did you prefer for each equation? Explain.

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PRACTISING

4. Solve each quadratic equation.
   a) $3x^2 + 5x = 9$
   b) $1.4x - 3.9x^2 = -2.7$
   c) $6x - 3 = 2x^2$
   d) $x^2 + 1 = x$

5. The roots for the quadratic equation
$$1.44a^2 + 2.88a - 21.6 = 0$$
are $a = 3$ and $a = -5$. Verify these roots.

6. Solve each equation. State the solutions as exact values.
   a) $3x^2 - 6x - 1 = 0$
   b) $x^2 + 8x + 3 = 0$
   c) $8x^2 + 8x - 1 = 0$
   d) $9x^2 - 12x - 1 = 0$

7. A student council is holding a raffle to raise money for a charity fund drive. The profit function for the raffle is
$$p(c) = -25c^2 + 500c - 350$$
where $p(c)$ is the profit and c is the price of each ticket, both in dollars.
   a) What ticket price will result in the student council breaking even on the raffle?
   b) What ticket price will raise the most money for the school's donation to charity?

Akpatok Island gets its name from the word *Akpat*, the Innu name for the birds that live on the cliffs.

8. Akpatok Island in Nunavut is surrounded by steep cliffs along the coast. The cliffs range in height from about 125 m to about 250 m.
   a) Suppose that someone accidentally dislodged a stone from a 125 m cliff. The height of the stone, $h(t)$, in metres, after t seconds can be represented by the following function:
$$h(t) = -4.9t^2 + 4t + 125$$
   How long would it take the stone dislodged from this height to reach the water below?
   b) Predict how much longer it would take for the stone to reach the water if it fell from a height of 250 m. Discuss this with a partner.
   c) The height of a stone, $h(t)$, in metres, falling from a 250 m cliff over time, t, in seconds, can be modelled by this function:
$$h(t) = -4.9t^2 + 4t + 250$$
   Determine how long it would take the stone to reach the water.
   d) How close was your prediction to your solution?

9. Keisha and Savannah used different methods to solve this equation:
$$116.64z^2 + 174.96z + 65.61 = 0$$
   a) Could one of these students have used factoring? Explain.
   b) Solve the equation using the method of your choice.
   c) Which method did you use? Why?

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10. The Moon's gravity affects the way that objects travel when they are thrown on the Moon. Suppose that you could throw a ball into the air from the top of a lunar module, 5.5 m high. The height of the ball, $h(t)$, in metres, over time, t, in seconds could be modelled by this function:

$$h(t) = -0.81t^2 + 5t + 6.5$$

a) How long would it take for the ball to hit the surface of the Moon?
b) If you threw the same ball from a model of the lunar module on Earth, the height of the ball could be modelled by this function:

$$h(t) = -4.9t^2 + 5t + 6.5$$

Compare the time that the ball would be in flight on Earth with the time that the ball would be in flight on the Moon.

Six lunar modules landed on the Moon from 1969 to 1972.

11. A landscaper is designing a rectangular garden, which will be 5.00 m wide by 6.25 m long. She has enough crushed rock to cover an area of 6.0 m^2 and wants to make a uniform border around the garden. How wide should the border be, if she wants to use all the crushed rock?

Closing

12. Discuss the quadratic formula with a partner. Make a list of everything you have both learned, from your work in this lesson, about using the quadratic formula to solve quadratic equations.

Extending

13. The two roots of any quadratic equation are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

a) Determine the sum of the roots of any quadratic equation.
b) Determine the product of the roots of any quadratic equation.
c) Solve the following quadratic equation:

$$10x^2 - 13x + 4 = 0$$

Determine the sum and the product of its roots.
d) Determine the sum and the product of the roots of the quadratic equation in part c), using your formulas from parts a) and b). Do your answers match your answers from part c)?
e) Determine the sum and the product of the solutions to questions 1 d), 2 a), 5, and 7.
f) How could you use your formulas from parts a) and b) to check your solutions to any quadratic equation?

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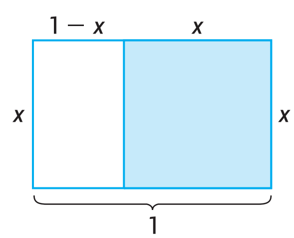
History Connection

The Golden Ratio

The golden ratio has been discovered and rediscovered by many civilizations. Its uses in architecture include the Great Pyramids in Egypt and the Parthenon in Greece. The golden ratio is the ratio of length to width in a rectangle with special properties, called the golden rectangle. This rectangle appears often in art, architecture, and photography.

The Manitoba Legislative Building



If you section off a square inside a golden rectangle so that the side length of the square equals the width of the golden rectangle, you will create a smaller rectangle with the same length : width ratio. Mathematicians sometimes use the Greek letter *phi*, φ, to represent this ratio.

To determine the golden ratio, you need to know that the ratio of length to width in the original rectangle, $\frac{1}{x}$, is equal to the ratio of length to width in the smaller rectangle, $\frac{x}{1-x}$.

A. Solve the following equation for x to determine the width of a rectangle with length 1. Then determine $\frac{1}{x}$ to get the golden ratio, φ.

$$\frac{1}{x} = \frac{x}{1-x}$$

B. Work with a partner or group to find golden rectangles in the photograph of the Manitoba Legislative Building.

C. Find more golden rectangles in architecture, art, and nature. Present your findings to the class.

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7 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How can I solve a quadratic equation by graphing?

A1: If the quadratic equation is in standard form, enter the corresponding function on a graphing calculator. Determine the x-intercepts of the parabola. These are the solutions to the equation.

A2: If the quadratic equation is not in standard form, you can graph the expressions on the left and right sides separately. The solutions to the equation are the x-coordinates of the points of intersection of the two functions.

Study Aid

- See Lesson 7.1, Examples 1 to 3.
- Try Mid-Chapter Review Questions 1 and 2.

Q: How can I solve a quadratic equation algebraically?

A: Write the equation in standard form:

$$ax^2 + bx + c = 0$$

Then determine the roots of the equation by factoring or by using the quadratic formula.

Study Aid

- See Lessons 7.2, Examples 1, 4, and 5, and 7.3, Examples 1, 3, and 4.
- Try Mid-Chapter Review Questions 5, 6, and 8.

<table>
<tr>
<th>Factoring</th>
<th>Using the quadratic formula</th>
</tr>
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<td>

If the expression $ax^2 + bx + c$ is factorable, then the equation $ax^2 + bx + c = 0$ is true when either of the factors is equal to 0.

For example:

$$2x^2 + 2x = 5x + 20$$
$$2x^2 - 3x - 20 = 0$$
$$(2x + 5)(x - 4) = 0$$

$2x + 5 = 0$ or $x - 4 = 0$

$2x = -5$ $x = 4$

$x = -\frac{5}{2}$

The roots are

$-\frac{5}{2}$ and 4

</td>
<td>

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For example:

$3x^2 - 4x - 5 = 0$

$a = 3$, $b = -4$, and $c = -5$

Substitute these values into the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{76}}{6}$$

The radicand is positive, so the equation has a solution.

$x = \frac{4 + \sqrt{76}}{6}$ or $x = \frac{4 - \sqrt{76}}{6}$

The roots are

$\frac{2 + \sqrt{19}}{3}$ and $\frac{2 - \sqrt{19}}{3}$

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PRACTISING

Lesson 7.1

1. Solve by graphing and determining the x-intercepts.

a) $0.5x^2 + 3x - 3.5 = 0$
b) $-3x^2 + 18x - 17 = 0$

2. Solve by graphing the expressions on both sides of the equation and determining the x-coordinates of the points of intersection.

a) $2x^2 - 6x = 5$
b) $-3x^2 + 4x = x^2 - 7$
c) $x(5 - 2x) = 3(x - 1)$
d) $5x - 0.25x^2 = 2(0.1x^2 - 3)$

3. If a skydiver jumps from an airplane and free falls for 828 m before he safely employs his parachute, his free fall could be modelled by the function

$$h(t) = -4.9t^2 + 10t + 828$$

where $h(t)$ is the height in metres and t is the time in seconds (ignoring air resistance). How long did this skydiver free fall?

Lesson 7.2

4. Rewrite each equation in standard form, and solve it by factoring. Verify each solution.

a) $x(x + 3) = 4$
b) $2z(z - 3) = -5(z - 9)$

5. Solve by factoring. Verify each solution.

a) $2x^2 + 5x - 3 = 0$
b) $36a^2 + 60a + 25 = 0$
c) $8c^2 - 26c + 15 = 0$
d) $1 - 8p + 16p^2 = 0$
e) $4t^2 - 81 = 0$
f) $9x^2 = 256$
g) $5w - 3w^2 = 0$
h) $7x = 3x^2$

6. Write a quadratic equation, in standard form, that has the roots $\frac{1}{2}$ and -6.

Lesson 7.3

7. Solve each equation by using the quadratic formula.

a) $3k^2 + 5k - 1 = 0$
b) $8n^2 + 15n + 6 = 0$
c) $35x^2 - 98x + 56 = 0$
d) $8y^2 + 90y + 187 = 0$

8. Solve each equation by graphing, by factoring, or by using the quadratic formula. Explain how you chose the method that you used.

a) $2p^2 + 11p + 12 = 0$
b) $3x(x - 4) = 2(5 - x^2)$
c) $12a^2 + 23a + 7 = 0$

9. An electronics company sells personal video recorders (PVRs) for \$189. At this price, the company sells 500 PVRs per day. The company wants to raise the price of the PVRs to increase its revenue. The revenue function is

$$r(d) = -300d^2 + 7165d + 94\,500$$

where $r(d)$ is the revenue, in dollars, and d is the number of \$1 price increases.

a) If the company wants to generate revenue of \$125 000 per day, how much will the price have to increase?
b) Is it possible for the company to earn revenue of \$140 000 per day by selling PVRs? Explain your reasoning.

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7.4 Solving Problems Using Quadratic Equations

GOAL

Analyze and solve problems that involve quadratic equations.

YOU WILL NEED

- graphing technology

EXPLORE...

- A right triangle has sides of length x, $2x + 4$, and $3x - 4$. Write a quadratic equation to determine the value of x. Is there more than one solution?

LEARN ABOUT the Math

The engineers who designed the Coal River Bridge on the Alaska Highway in British Columbia used a supporting arch with twin metal arcs.

The function that describes the arch is

$$h(x) = -0.005\,061x^2 + 0.499\,015x$$

where $h(x)$ is the height, in metres, of the arch above the ice at any distance, x, in metres, from one end of the bridge.

? How can you use the width of the arch to determine the height of the bridge?

EXAMPLE 1 Solving a problem by factoring a quadratic equation

Determine the distance between the bases of the arch. Then determine the maximum height of the arch, to the nearest tenth of a metre.

Morgan's Solution

The coordinates of the maximum are (x, y), where x is halfway between the two bases of the arch and y is the height of the arch.

I reasoned that the bridge is symmetrical and resting on the vertex of the arch.

$$h(x) = -0.005\,061x^2 + 0.499\,015x$$
$$0 = -0.005\,061x^2 + 0.499\,015x$$

I wrote an equation to determine the x-coordinates of the bases of the arch. The height at each base is 0 m, so the value of $h(x)$ at these points is 0.

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$0 = -5.061x^2 + 499.015x$
$0 = x(-5.061x + 499.015)$

I multiplied both sides by 1000 and factored the equation.

$x = 0$ or $-5.061x + 499.015 = 0$
$-5.061x = -499.015$
$x = 98.600...$

I solved the equation.

One base is at 0 m, and the other is at 98.600... m. The width of the arch is 98.600... m.

The width of the arch is the distance between the two bases.

Equation of axis of symmetry:

$$x = \frac{0 + 98.600...}{2}$$

$x = 49.300...$

The x-coordinate of the vertex is 49.3...

The function is quadratic, so the arch is a parabola with an axis of symmetry that passes through the vertex.

$h(x) = -0.005\,061x^2 + 0.499\,015x$

For $x = 49.300...$,

$y = -0.005\,061\,(49.300...)^2 + 0.499\,015\,(49.300\,...)$

$y = -12.300... + 24.601...$

$y = 12.300...$ m

The height of the arch is the y-coordinate of the vertex of the parabola.

The distance between the bases of the bridge is 98.6 m. The height of the arch above the ice is 12.3 m.

Reflecting

A. How did determining the x-coordinates of the bases of the arch help Morgan determine the height of the arch?

B. What reasoning might have led Morgan to multiply both sides of the equation by 1000?

C. How did Morgan know that the equation $0 = -5.061x^2 + 499.015x$ could be factored?

D. How else could Morgan have solved her quadratic equation?

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APPLY the Math

EXAMPLE 2 Solving a number problem by graphing

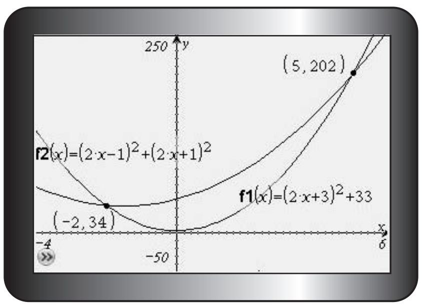
Determine three consecutive odd integers, if the square of the largest integer is 33 less than the sum of the squares of the two smaller integers.

Hailey's Solution

Let the three integers be $2x - 1$, $2x + 1$, and $2x + 3$.

$$(2x + 3)^2 + 33 = (2x - 1)^2 + (2x + 1)^2$$

> Odd numbers are not divisible by 2. According to the problem, if I add 33 to the square of the largest number, my result will equal the sum of the squares of the two smaller numbers.



> I graphed both sides of my equation and determined the points of intersection.

The points of intersection are $(-2, 34)$ and $(5, 202)$. The two possible values of x are -2 and 5.

If $x = -2$,

$2x - 1$	$2x + 1$	$2x + 3$
$2(-2) - 1$	$2(-2) + 1$	$2(-2) + 3$
-5	-3	-1

The integers are -5, -3, and -1.

> I determined three consecutive odd integers for each value of x.

If $x = 5$,

$2x - 1$	$2x + 1$	$2x + 3$
$2(5) - 1$	$2(5) + 1$	$2(5) + 3$
9	11	13

The integers are 9, 11, and 13.
The consecutive odd integers could be -5, -3, and -1, or they could be 9, 11, and 13.

> My answers seem reasonable. $13^2 = 169$ and this is 33 less than $9^2 + 11^2$, which is 202. $(-1)^2 = 1$ and this is 33 less than $(-5)^2 + (-3)^3$, which is 34.

Your Turn

Why was Hailey's method better for solving the problem than simply guessing and testing numbers?

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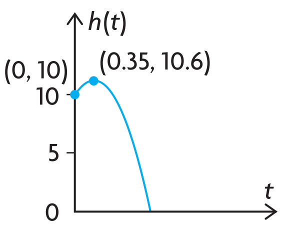
EXAMPLE 3 Solving a problem by creating a quadratic model

Synchronized divers perform matching dives from opposite sides of a platform that is 10 m high. If two divers reached their maximum height of 0.6 m above the platform after 0.35 s, how long did it take them to reach the water?

Canadians Émilie Heymans and Blythe Hartley won bronze medals at the 2004 Olympic Games.

Oliver's Solution

Let t represent the time in seconds.
Let $h(t)$ represent the height in metres over time.



I sketched a graph to show how the divers' height changed as time passed. I knew that the vertex of the parabola was (0.35, 10.6) because the maximum height of 10.6 m (0.6 m above the 10 m platform) was attained after 0.35 s.

$$h(t) = a(t - 0.35)^2 + 10.6$$

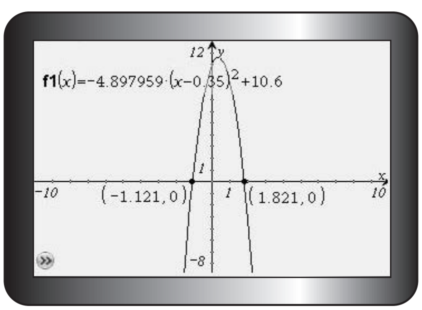
I wrote a quadratic function in vertex form.

$$\begin{aligned} 10 &= a(0 - 0.35)^2 + 10.6 \\ 10 &= a(-0.35)^2 + 10.6 \\ 10 &= 0.1225a + 10.6 \\ -0.6 &= 0.1225a \\ -4.897\ldots &= a \end{aligned}$$

The platform is 10 m high. Therefore, when $t = 0$, $h(t) = 10$. I substituted these values into my equation and solved for a.

$$f(x) = -4.897\ldots(x - 0.35)^2 + 10.6$$

I wrote a function to represent the dive. I knew that the height would be 0 when the divers hit the water.



I graphed my function and determined the x-intercepts.

The zeros of my function are -1.121 and 1.821.
The solution -1.121 s is inadmissible.

Time cannot be negative in this situation.

The divers reached the water after about 1.821 s.

Your Turn

How does Oliver's first graph show that there is only one solution to the problem?

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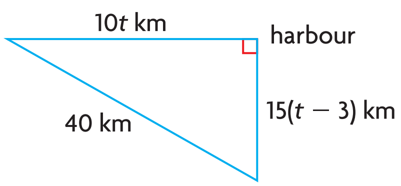
EXAMPLE 4 Visualizing a quadratic relationship

At noon, a sailboat leaves a harbour on Vancouver Island and travels due west at 10 km/h. Three hours later, another sailboat leaves the same harbour and travels due south at 15 km/h. At what time, to the nearest minute, will the sailboats be 40 km apart?

Nikki's Solution

Let t be the number of hours it will take for the sailboats to be 40 km apart.



I drew a diagram to show the paths of the two sailboats. They are sailing at right angles to each other.

The first boat travels t hours at 10 km/h.

The second boat leaves 3 h later, so it travels for $t - 3$ h at 15 km/h.

$$(10t)^2 + [15(t-3)]^2 = 40^2$$

I used the Pythagorean theorem to write an equation that relates the distances travelled to the 40 km distance between them.

$$\begin{aligned}(10t)^2 + (15t - 45)^2 &= 40^2\\ 100t^2 + 225t^2 - 1350t + 2025 &= 1600\\ 325t^2 - 1350t + 425 &= 0\\ 13t^2 - 54t + 17 &= 0\end{aligned}$$

I simplified my quadratic equation and wrote it in standard form. Then I divided both sides by 25.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-54) \pm \sqrt{(-54)^2 - 4(13)(17)}}{2(13)}$$

$$t = \frac{54 \pm \sqrt{2032}}{26}$$

$$t = 0.343\ldots \quad \text{or} \quad t = 3.810\ldots$$

I used the quadratic formula to solve for t, the number of hours that it will take for the boats to be 40 km apart.

The solution 0.343... h is inadmissible.
(0.81... h)(60 min/1 h) = 48.6 min
The boats will be 40 km apart at 3:49 p.m.

The boats could not be 40 km apart after 0.343... h, because the second boat has not yet left the harbour and the first boat is less than 10 km out.

Your Turn

a) Tomas solved the same problem. However, he used t to represent the time for the second boat's journey. How would the labels on Tomas's diagram be different from the labels on Nikki's diagram?

b) Use Tomas's method to solve the problem.

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In Summary

Key Ideas

- A function, a graph, or a table of values can represent a relation. Use the form that is most helpful for the context of the problem.
- Depending on the information that is given in a problem, you can use a quadratic function in vertex form or in standard form to model the situation.

Need to Know

- A problem may have only one admissible solution, even though the quadratic equation that is used to represent the problem has two real solutions. When you solve a quadratic equation, verify that your solutions make sense in the context of the problem.

CHECK Your Understanding

1. The engineers who built the Coal River Bridge on the Alaska Highway in British Columbia used scaffolding during construction. At one point, scaffolding that was 9 m tall was placed under the arch. The arch is modelled by the function

$$h(x) = -0.005\,061x^2 + 0.499\,015x$$

a) Describe a strategy you could use to determine the minimum distance of this scaffolding from each base of the arch.
b) Use your strategy from part a) to solve the problem.
c) Compare your strategy and solutions with a classmate's strategy and solutions. What other strategies could you have used?

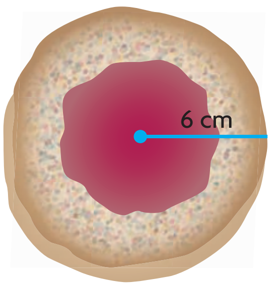
Communication Tip

The formula $V = \pi r^2 h$ can also be written as a quadratic function:
$f(r) = \pi r^2 h - V$

PRACTISING

2. A company manufactures aluminum cans. One customer places an order for cans that must be 18 cm high, with a volume of 1150 cm^3.
a) Use the formula $V = \pi r^2 h$ to determine the radius that the company should use to manufacture these cans.
b) Graph the function that corresponds to $0 = \pi r^2 h - V$ to determine the radius.
c) Which method do you prefer? Explain why.

3. The sum of two numbers is 11. Their product is -152. What are the numbers?



4. A doughnut store sells doughnuts with jam centres. The baker wants the area of the jam to be about equal to the area of the cake part of the doughnut, as seen from the top. The outer radius of a whole doughnut is 6 cm. Determine the radius of the jam centre.

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5. Duncan dives with a junior swim club. In a dive off a 7.5 m platform, he reaches a maximum height of 7.94 m after 0.30 s. How long does it take him to reach the water?

6. A jet skier leaves a dock at 8 a.m. and travels due west at 36 km/h. A second jet skier leaves the same dock 10 min later and travels due south at 44 km/h. At what time of day, to the nearest minute, will the two jet skis be 20 km apart?

7. Alexis sells chocolate mousse tortes for $25. At this price, she can sell 200 tortes every week. She wants to increase her earnings, but, from her research, she knows that she will sell 5 fewer tortes per week for each price increase of $1.
   a) What function, $E(x)$, can be used to model Alexis's earnings, if x represents the price increase in dollars?
   b) What higher price would let Alexis earn the same amount of money she earns now?
   c) What should Alexis charge for her tortes if she wants to earn the maximum amount of money?

8. Two consecutive integers are squared. The sum of these squares is 365. What are the integers?

9. Brianne is a photographer in southern Alberta. She is assembling a display of photographs of endangered local wildlife. She wants each photograph in her display to be square, and she wants the matte surrounding each photograph to be 6 cm wide. She also wants the area of the matte to be equal to the area of the photograph itself. What should the dimensions of each photograph be, to the nearest tenth of a centimetre?

Closing

10. Quadratic equations that describe problem situations are sometimes complicated. What are some methods you can use to simplify these equations and make them easier to solve?

Extending

11. Aldrin and Jan are standing at the edge of a huge field. At 2:00 p.m., Aldrin begins to walk along a straight path at a speed of 3 km/h. Two hours later, Jan takes a straight path at a 60° angle to Aldrin's path, walking at 5 km/h. At what time will the two friends be 13 km apart?

12. Frances is an artist. She wants the area of the matte around her new painting to be twice the area of the painting itself. The matte that she wants to use is available in only one width. The outside dimensions of the same matte around another painting are 80 cm by 60 cm. What is the width of the matte?

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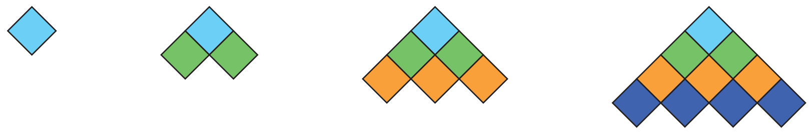
Applying Problem-Solving Strategies

Determining Quadratic Patterns

Many geometric patterns have connections to algebra. Examining a pattern can help you develop a formula that describes the general rule for the pattern.

The Puzzle

This pattern grows as a new row of tiles is added to each figure.



How many tiles would you need to construct a figure with 12 rows?

The Strategy

A. Copy this table.

Number of Rows	0	1	2	3	4			
Number of Tiles in Bottom Row	0	1	2	3				
Total Number of Tiles	0	1	3	6				

B. Complete your table for the next three figures in the pattern above.

C. Explain how you would determine the total number of tiles in figures with 8, 9, and 10 rows.

D. Write a quadratic equation that gives the total number of tiles in a figure with any number of rows.

E. Test your equation by using it to determine the total number of tiles in a figure with 12 rows. Check your answer by extending your table.

F. When you developed your equation for the pattern, did you use inductive or deductive reasoning? Explain.

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7 Chapter Self-Test

1. The acceleration due to gravity on Mars is 3.8 m/s^2. Suppose that a rocket is launched on Mars, with an initial velocity of 64 m/s^2. The height of the rocket, $h(t)$, in metres, after t seconds can be modelled by the following function:

$$h(t) = -\frac{1}{2}(3.8)t^2 + 64t$$

a) Graph the function. How long will the flight last?
b) At what time will the rocket reach a height of 400 m?
c) Will the rocket reach a height of 550 m? Explain.

2. Solve the following equation by graphing the expressions on both sides of the equation:

$$(2x - 7)(x + 2) = (3 - x)(1 + 4x)$$

3. Solve by factoring. Verify each solution.
a) $81y^2 - 625 = 0$
b) $12z - 6z^2 = 0$
c) $3c^2 - 48 = 0$
d) $5h^2 = 4h$

4. Solve by factoring. Verify each solution.
a) $x^2 + 11x + 24 = 0$
b) $8a^2 + 31a - 4 = 0$
c) $5c = c^2 - 6$
d) $25x^2 + 10x + 5 = 5x^2 - 3x + 3$

5. Solve by using the quadratic formula.
a) $x^2 + 5x - 8 = 0$
b) $4x^2 - 12x - 3 = 0$
c) $0.25x^2 - 0.3x + 0.09 = 0$
d) $5x^2 + 6x + 7 = 0$

6. Determine three consecutive even numbers, if the square of the largest number less the square of the middle number is 20 less than the square of the smallest number.

7. The Yukon Bridge is a suspension bridge with a parabolic shape. Its height, $h(t)$, in metres, can be represented by the equation

$$h(t) = 0.005\,066w^2 - 0.284\,698w$$

where the height is 0 m at the endpoints and w is the length of a straight line from one endpoint to the other.
a) Determine the length of line w.
b) What is the maximum drop in height from line w to the bridge?

The Yukon Bridge spans the Tutshi River in northern British Columbia. To build the Yukon Bridge, engineers had to rig a temporary skyline. This skyline was used to transfer drilling equipment and to anchor rods to the far side of the river. A helicopter was used to install the support towers on the far side of the river.

8. A rectangle is 5 cm longer than it is wide. The length of the diagonal of the rectangle is 18 cm. Determine the dimensions of the rectangle.

WHAT DO You Think Now? Revisit **What Do You Think?** on page 395. How have your answers and explanations changed?

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7 Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 7.4, Examples 1 to 4.
- Try Chapter Review Questions 5 to 10.

Q. What strategies can you use to solve contextual problems that involve quadratic equations?

A: Problems that involve quadratic equations can be solved with or without graphing technology.

If you don't have access to graphing technology, you can use these strategies:

- Express the equation in standard form:

$$ax^2 + bx + c = 0, a \neq 0$$

  Try to factor the expression $ax^2 + bx + c$. If it is factorable, set each factor equal to zero and solve the resulting linear equations.

- For equations that are not factorable, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  The quadratic formula can always be used to solve quadratic equations that have solutions.

If you have access to graphing technology, you can use these strategies:

- Express the equation in standard form:

$$ax^2 + bx + c = 0, a \neq 0$$

  Graph the corresponding function:

$$f(x) = ax^2 + bx + c$$

  Locate the zeros of the function. These are the x-intercepts of the graph of the function. The values of the x-intercepts are the solutions or roots of the quadratic equation.

- If the equation is not expressed in standard form, you can graph the left and right sides of the equation by treating each side as a function. The solutions to the equation are the x-coordinates of the points of intersection of the two functions.

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Q. When solving contextual problems that involve quadratic equations, will the solution(s) to the equation always be solutions to the problem?

A: No. Often the context of the problem requires that restrictions be placed on the independent variable in the function modelling the situation. If a solution does not lie within the restricted domain of the function, then it is not a solution to the problem. Such solutions are called inadmissible.

For example, consider this problem:

Sylvia dives from a tower whose platform is 10 m above the surface of the water. Her dive can be modelled by the function

$$h(t) = -4.9t^2 + 1.5t + 10$$

where $h(t)$ represents her height above the water, in metres, and t represents time from the start of her dive, in seconds. How long does it take for Sylvia to enter the water, to the nearest tenth of a second?

$0 = -4.9t^2 + 1.5t + 10$	The diver's height is 0 m when she enters the water.
$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $t = \dfrac{-1.5 \pm \sqrt{(1.5)^2 - 4(-4.9)(10)}}{2(-4.9)}$ $t = \dfrac{-1.5 \pm \sqrt{198.25}}{-9.8}$ $t = \dfrac{-1.5 \pm 14.080...}{-9.8}$	Solve the quadratic equation using the quadratic formula.
$t = -1.283...$ or $t = 1.589...$ In this case, $t = -1.283...$ is an inadmissible solution. Sylvia takes 1.6 s to enter the water.	The domain of $h(t)$ is $t \geq 0$, where $t \in \mathrm{R}$, since time must be positive in the context.

Study Aid

- See Lesson 7.4, Examples 1 to 4.
- Try Chapter Review Questions 5 to 10.

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PRACTISING

Lesson 7.1

1. Solve by graphing.
a) $6x^2 - 13x + 6 = 0$
b) $64x^2 + 112x + 49 = 0$
c) $-5x^2 - 8x + 3 = 0$
d) $-0.25x^2 + 2x + 5 = 0$

2. Solve by graphing.
a) $3t - t^2 = -6$
b) $4n^2 + 1 = n + 3$
c) $b(9 - 3b) + 7 = 2(b - 5) + (0.5)b^2$
d) $c^2 - 38c + 340 = 3c^2 - 96c + 740$

Lesson 7.2

3. Solve by factoring. Verify each solution.
a) $s^2 - 7s - 60 = 0$
b) $10x^2 + 17x - 20 = 0$
c) $2a^2 + 10a + 12 = 0$
d) $-3x^2 - 5x + 2 = 0$
e) $16d^2 - 169 = 0$
f) $8r - 3r^2 = 0$
g) $3x^2 - 2x = 81 - 2x - x^2$
h) $4(m^2 - 4m + 6) = 3(2m^2 + 8)$

Lesson 7.3

4. Solve by using the quadratic formula.
a) $117x^2 - 307x + 176 = 0$
b) $f^2 + 2f - 2 = 0$
c) $7h^2 + 6h = 5$
d) $6x^2 + 8x + 4 = 0$

Lesson 7.4

5. Determine three consecutive positive odd integers, if the sum of the squares of the first two integers is 15 less than the square of the third integer.

6. A right triangle has a perimeter of 120 cm. One side of the triangle is 24 cm long. Determine the length of the other side and the length of the hypotenuse.

7. A fishing boat leaves a dock at noon and travels due west at 40 km/h. A second boat leaves the same dock 20 min later and travels due south at 51 km/h. At what time, to the nearest minute, will the two boats be 116 km apart?

8. A skydiver jumps out of an airplane at an altitude of 3.5 km. The altitude of the skydiver, $H(t)$, in metres, over time, t, in seconds, can be modelled by the function

$$H(t) = 3500 - 5t^2$$

a) How far has the skydiver fallen after 10 s?
b) The skydiver opens her parachute at an altitude of 1000 m. How long did she free fall?

9. Two integers differ by 12. The sum of the squares of the integers is 1040. Determine the integers.

10. Tickets to a school dance cost \$5. The projected attendance is 300 people. The dance committee projects that for every \$0.50 increase in the ticket price, attendance will decrease by 20. What ticket price will generate \$1562.50 in revenue?

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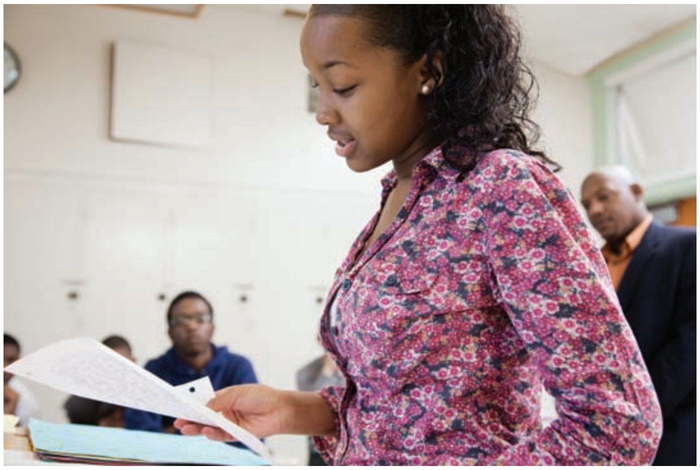
7 Chapter Task

A Teaching Tool

Have you ever heard the following saying?
"To teach is to learn twice."
For this task, you will illustrate and explain what you have learned about quadratic equations, using a format of your choice.



? How do you solve a quadratic equation?

A. Choose a format for your presentation. Here are some possibilities:
- short story
- song
- poem or rap
- multimedia presentation
- mobile
- flow chart
- T-shirt design

B. In your presentation, explain the three methods that can be used to solve a quadratic equation. Determine the important concepts to explain for each method.

C. Write a quadratic equation, and show how to solve it using the method you think is best. Explain your solution, including why you chose the method you did.

Task Checklist

✔ Are your explanations clear?

✔ Did you use appropriate mathematical language?

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7 Project Connection

The Final Product and Presentation

Your final presentation should be more than just a factual written report of the information you have found. To make the most of your hard work, select a format for your final presentation that will suit your strengths, as well as your topic.

Presentation Styles

To make your presentation interesting, use a format that suits your own style. Here are some ideas:

- a report on an experiment or an investigation
- a summary of a newspaper article or a case study
- a short story, musical performance, or play
- a web page
- a slide show, multimedia presentation, or video
- a debate
- an advertising campaign or pamphlet
- a demonstration or the teaching of a lesson

Here are some decisions that other students have made about the format for their presentation:

Project 1: Weather Predictions
Muhamud has researched the mathematics of weather predictions. He has decided to make his presentation a demonstration of how a weather report is prepared, including the mathematics used, followed by an actual television weather report. He plans to submit a written report on his research and conclusions, as well.

Project 2: Gender Differences
Ming has studied the differences between the responses of females and males on cognitive aptitude tests. To illustrate her findings, she will have the class complete one of the assessment tasks during her presentation and then compare the results with standardized norms. In her report, Ming plans to include testing she has done on randomly selected students at her school.

Executive Summary

Sometimes, it is effective to give your audience an executive summary of your presentation. This is a one-page summary of your presentation, which includes your research question and the conclusions you have made. Ask your teacher about making copies of your summary for the class.

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Project Example — Creating Your Presentation

Sarah chose the changes in population of the Western provinces and the territories over the last century as her topic. Below, she describes how she determined which format to use for her presentation.

Sarah's Presentation

Because most of my supporting information is graphical, I am going to use a multimedia slide show. I will include some tables and graphs to show that the population of British Columbia and Alberta grew faster than the population of the rest of the Western and Northern provinces and territories. I will give all of the audience members an executive summary of my research, which will include my research question, my data (with the necessary supporting visuals), and my conclusions. I will give my teacher the full report.

Evaluating Your Own Presentation

Before giving your presentation, you can use these questions to decide if your presentation will be effective:

- Did I define my topic well? What is the best way to define my topic?
- Is my presentation focused? Will my classmates find it focused?
- Did I organize my information effectively? Is it obvious that I am following a plan in my presentation?
- Am I satisfied with my presentation? What might make it more effective?
- What unanswered questions might my audience have?

Your Turn

A. Does your topic suit some presentation formats better than others? Explain why.

B. From which presentation format do you think your audience will gain the greatest understanding? Why?

C. Choose a format for your presentation, and create your presentation.

D. Use the questions provided in Evaluating Your Own Presentation to assess your presentation. Make any changes that you think are needed, as a result of your evaluation.

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